90-00137/

Forest Research Report No. 123

# Dimensional Growth, Difference-Based Equations and Growth Prediction

F.L. Raymond

Ontario Forest Research Institute
Ministry of Natural Resources
P.O. Box 7400
Maple, Ontario L6A 1S9
(416) 832-2761

Ontario Ministry of Natural Resources

Lyn McLeod
Minister

**Canadian Cataloguing in Publication Data**

Raymond, F.L.
Dimensional growth, difference-based equations and growth prediction

(Forest research report, ISSN 0381-3924 ; 123)
Bibliography: p.
ISBN 0-7729-5748-7

1. Trees--Growth--Mathematical models.
I. Ontario Forest Research Institute
II. Title. III. Series.

SD396.R39 1989 634.9'285 C89-099636-9

© 1989, Queen's Printer for Ontario

Printed in Ontario, Canada

Single copies of this publication
are available at no charge from
the address noted below.

Ministry of Natural Resources
Ontario Forest Research Institute
P.O. Box 7400
Maple, Ontario L6A 1S9

(416) 832-2761

RAYMOND, F.L. 1989. Dimensional growth, difference-based equations and growth prediction. Ontario Ministry of Natural Resources, Forest Research Report No. 123. 19 p.

## ABSTRACT

Searching for equations to fit the typically fluctuating growth rates in various dimensions of essentially mature trees disclosed that non-constant growth rates require difference-based equations rather than differential-based equations. The simplest difference-based equation described relationships between increments and growth rates exactly without indicating trends. When a coefficient reflecting the general trend of growth rates decreasing over time was added to the equation, fitting either individual trees or aggregations of trees tended to follow the irregular growth pathways for the period fitted. This tendency appears to have been enhanced by the use of uncorrected sums of squares for regression. Offsetting the general trend of decreasing rates were: occasional years possibly associated with good climate, and sequences of years apparently responding to disappearances of competing neighbors. Over periods of five or 10 years, applying fitted coefficients plus the initial growth value usually produced good to excellent results, but projections from them tended not to follow subsequent growth for long. Correctable or dynamic predictions based on dynamic equations responsive to changing growth rates appear to offer the greatest promise for satisfactory prediction of future growth.

## TABLE OF CONTENTS


INTRODUCTION ..... 1

MATERIALS AND METHODS ..... 2

RESULTS ..... 4

DISCUSSION ..... 7

PROGRAM AVAILABILITY ..... 10

ACKNOWLEDGMENTS ..... 11

REFERENCES ..... 11


## APPENDICES


Table 1 — Growth values, rates and differences for diameters outside bark at breast height, for the hard maple tree. 12

Table 2 — Results of long-period fittings of dbhob growth in the jack pine tree. 14

Figure 1 — Plotted b-values for the jack pine tree, as in [2]. (A) is dbhob, (B) is total tree height, (C) is gross total volume, (D) is bole cambial area, (E) is gross merchantable volume and (F) is basal area. 15

Figure 2 — Growth curves for a 131-year-old codominant hard maple tree, consisting of observed values, o, and fitted curves, --, from [3]. (A) is dbhob, (B) is total tree height, (C) is gross total volume, (D) is bole cambial area and (E) is gross merchantable volume. 16

Figure 3 — Growth curves for a 69-year-old dominant jack pine tree, consisting of observed values, o, and fitted curves, --, from [3]. (A) is dbhob, (B) is total tree height, (C) is gross total volume, (D) is bole cambial area and (E) is gross merchantable volume. 17

Figure 4 — Growth curves for a 97-year-old codominant black spruce tree, consisting of observed values, o, and fitted curves, --, from [3]. (A) is dbhob, (B) is total tree height, (C) is gross total volume, (D) is bole cambial area and (E) is gross merchantable volume. 18

Figure 5 — Growth layer thickness profiles for the last 44 years of growth in the black spruce tree. (A) is for the years 1964 through 1985 and (B) is for the years 1942 through 1963. 19

# INTRODUCTION

Several years of peripheral association with the Tree Ring Increment Measurement (TRIM) system, during and after its development by Fayle and MacIver (1986), convinced me of the need to obtain models capable of "tracking" or fitting the typically irregular growth curves obtained for various dimensions of trees. TRIM supports precise measurement of bark thickness and of tree ring widths along selected radii of disks cut from the bole of a tree. Then it reconstructs the bole as an accumulation of the comprised individual growth layers. It also expects that all the trees in each plot be described and mapped so that growth can be considered on a unit-area basis. Any desired subset of the sampled trees from various plots can be aggregated into a "giant" bole. Such values may be converted into means representing average trees, if desired. Growth in each of several dimensions of any bole can be graphed or plotted in various ways. The growth curves often reflect years or longer periods with rates that fluctuate about the general path.

Preliminary examination of such curves suggested that a study of growth rates might prove more productive than one confined to growth increments. Early efforts consisted of exploring simple elaborations of a basic equation involving growth rates. Fitting those elaborations by means of nonlinear regression produced some interesting but biased results.

Eventually it became clear that the basic growth-rate equation, which was an integral solution of a "differential" equation, or any elaboration of it, was unsuitable for situations in which the growth rate does not remain constant. Dimensional growth of living organisms probably never proceeds at anything like a constant rate; certainly that in forest trees does not.

This problem was resolved by replacing the differential-based equations with difference-based equations related to "difference" equations. Difference-based equations are capable of providing exact fits in situations involving rates that constantly increase or decrease. They also tend to track irregular growth fairly well even when the growth rates change nearly every year.

A very condensed version of this paper, entitled "Fitting, Modeling and Predicting Dimensional Growth In Trees", was presented at the IUFRO Conference on Forest Simulation Systems in Berkeley, California on November 3, 1988.

# MATERIALS AND METHODS

The tree data utilized in this study were obtained from copies of files generated during investigations by other people. Most of those files had been created using the TRIM system described by Fayle and MacIver (1986) and by MacIver, Raymond and Masarovich (1988).

The TRIM system accepts radial measurements of disks cut from tree boles, together with records of the height of each disk above the ground and of total tree height. For every disk in any tree, bark thickness, ring- and pith-widths are measured along selected radii. There are three application phases.

In its first application phase, the TRIM system stores each measurement from all radii of every disk, together with the counts of those measurements, in a single file. Disk heights and the address of the beginning of each subset of radial measurements are stored in a separate file. After inspection and any necessary correction, the first file is replaced with one containing a single set of averaged radial measurements for each disk. The address file is updated to also contain the address for each set of averaged data.

In its second phase, TRIM either accepts or interpolates values for total height at the end of each growing season. Then TRIM combines the ring-width data with the disk- and tree-heights into estimates of diameters, areas and volumes of wood for each year of growth or growth layer. As each new kind of estimate is generated, the results are stored in a new file. Finally, summarized results are stored in yet another file.

The third phase of TRIM provides the reporting on various sets of data by either printing tables or plotting graphs.

The software of the TRIM system and most of the files it produces are system specific to Apple II+ or Apple IIe microcomputers. Because those systems do not readily provide the degree of mathematical precision desired for this study, only the address file and the averaged-data file for any tree were copied onto floppy disks compatible with the DEC/Pro 380 microcomputer.[1]

Data were first obtained for 20 hard (sugar) maple (*Acer saccharum* Marsh.) from nine locations in the central part of Algonquin Park. Sixteen of them, ranging in "age" from 55 to 178 years, had been processed on a TRIM system. Age is in quotes because it is determined from the lowermost disks which were taken at 10 cm above ground level. Disks were cut at 40-cm intervals from 40 cm above ground level to near the top of the tree; additional disks were taken at 10 cm, 130 cm (breast height) and at the base of the live crown.

The other four hard maple trees had been processed on a TRIM-like system. They ranged in "age" from 145 to 300 years. Disks were cut at 1-m intervals from 1 m above the ground to near the top. An additional disk had been taken at 35 cm.

Although the data for those 20 trees were not ideal for present purposes because the data sets for gross total volume and cambial area in each tree were "censored" in that no disks had been taken at ground level, they contained what appeared to be clear evidence of occasional partial releases resulting from the disappearances of neighboring trees.

In an attempt to confirm such evidence, similar files were obtained for 27 jack pine (*Pinus Banksiana* Lamb.) trees from three randomly located plots near Kirkland Lake, Ontario. The data came from the destructive sampling of all living trees within the central 10 m by 10 m portions of 20 m by 20 m plots in a stand that clearly appeared to have been of fire origin. One of those plots contained three standing dead trees at the time of sampling. Twenty-two of those 27 trees had disks taken at ground level and ranged in age from 64 to 71 years. The living trees generally had disks cut at 15-cm intervals from ground level to 120 cm and from 2.05 m below the top to 5 cm from the top. Additional disks were taken at 130 cm, at the base of the live crown and at 50-cm intervals from 150 cm to a point within 50 cm of the uppermost 2 m. The disk sampling intervals approximate the recommendations of MacIver, Raymond and Masarovich (1988).

Finally, as a check on the results already found, similar files for a black spruce (*Picea mariana* (Mill.) B.S.P.) tree grown in a swampy area north of Cochrane, Ontario were obtained and processed. This tree was 97 years old. Disks had been taken at 15-cm intervals from ground level to 120 cm and in the uppermost 2.05 m; in between, disks were taken at 30-cm intervals, with additional disks taken at 130 cm and at the base of the live crown. This tree was located about five meters from a drainage ditch believed created by W. Stanek, in 1961, using dynamite. The stand contained a mixture of black spruce and tamarack (*Larix laricina* (Du Roi) Koch) and was believed to have undergone considerable spruce budworm (*Choristoneura fumiferana* Clem) damage prior to 1950.[2]

All of the disks from the above-mentioned trees were measured along two radii: north and south.

Development of TRIM software on the DEC 380 for the three phases of TRIM had largely been completed before this study began, as had a program for nonlinear regression. The DEC version of TRIM software contains certain features and options not in the Apple version. The enhancements include some additional checking for errors, some optional calculations and some additional routine calculations. It routinely produces values for diameter, breast height, outside bark

---

[1]This does not constitute an endorsement.

[2]Personal communication, D.C. MacIver.

(dbhob). If no disk had been taken at breast height, inside-bark values are interpolated for that height and used to generate dbhob values by adding interpolated bark thicknesses in proportion to their magnitude. Then the dbhob values and their basal area counterparts are stored in the summary file. The DEC version also automatically calculates gross merchantable volume for all stems large enough to have any. Stump height is assumed to be 30 cm. Top height is assumed to be at a diameter of 10 cm outside bark for conifers or of 22 cm for hardwoods. The results are stored in a second summary file.

The Apple version of TRIM utilizes calculations for gross total volume which assume that the stem is a paraboloid throughout its length. Demaerschalk and Kozak (1977) pointed out that, "Most textbooks emphasize that the tree stem is composed of two to three different forms, with the butt approximating a neiloid, the central portion approaching the frustum of a paraboloid, and the top portion similar to a paraboloid or a cone." By treating the entire bole as being paraboloid, the Apple calculations are biased upward wherever the profile is conic and even more biased wherever the profile is neiloid. Such biases can be reduced by keeping the distances between disks very small. The DEC version offers the option of assuming a profile represented by conic frustums throughout the stem to partially offset the biases. Both versions store all of their results in single precision arithmetic.

Because disks are occasionally cut very close to branch whorls or nodes, portions of the resulting stem profiles can be irregular. An option to exercise gentle smoothing of those profiles by means of three-point moving averages was provided by the DEC version. However, no smoothing of any kind was employed in the present study. The conic option for volume calculations was utilized throughout.

The solution procedure in the DEC program for nonlinear regression was generalized from a program for a specific application of the Marquardt Algorithm to an equation with three coefficients. A listing of the program, labelled "ZOOL 202", was obtained from the University of Toronto in 1967. Tests of the generalized solution procedure demonstrated that it produced results identical to those obtained using the matrix inversion procedure Grosenbaugh (1967) employed in his REX package.

Both regression programs utilize the linearization of any nonlinear equation by replacing it with its partial differentials, as was discussed by Draper and Smith (1966). Then the efficiency of the iterative solution-procedure is facilitated by applying ridge-regression factors to the main diagonal of the variance-covariance matrix. During each iteration, both programs successively employ several different rates at which those ridging factors are altered. Both are written in Fortran using double-precision arithmetic throughout.

There is a fundamental difference in application of the two programs. The ZOOL 202 program corrects the total sum of squares and the residual sum of squares for the mean before testing for goodness of fit, even though there appears not to have been any intention of utilizing the corrected values to generate an additional coefficient value. The DEC program uses uncorrected sums of squares throughout, both because no added coefficient was desired and because applying the corrections often led to unsatisfactory regression values at times when the tracking was quite satisfactory.

After calculating coefficient values for any chosen period of time by fitting them to the growth rates of the observed growth values, the DEC program generates two different tables of residuals. In the first table, the fitted estimates of growth are compared to the observed values of growth. In the second table, new estimates are obtained by assuming that the fitted coefficient values remained constant over the time period and those growth estimates are also compared to the observed growth values.

For each dimension of all trees studied, a file such as is illustrated in Table 1, for growth in dbhob of a hard maple tree, was produced. This file, containing more significant digits than

Table 1, enabled visual examination and provided the data needed for both the regressions and the resulting tables of residuals.

Standard fitting practice was to first "scan" through the data by fitting essentially successive five-year periods. Except when n-1 was a multiple of five, this meant some overlapping of the first two periods, in order that the last period coincides with the last five years of growth. The minimum period-length of five years was chosen to satisfy the Belsley, Kuh and Welsch (1980) "rule of thumb" that the number of coefficients estimated should not exceed 40% of the observations. Next a second scan was performed as follows: first do a fit over the period from 1 to n-1 (last year of growth), then fit a sequence of periods that, except for any overlap, is each 10 years shorter than the previous one and that all end with the last 10 years of growth.

In an attempt to improve upon the limited capability to predict growth using coefficients fitted over long periods, the regression program was further modified to accommodate weighted fitting. (Cf. Cunia (1964), Draper and Smith (1966) and Raymond (1980) on weighted regression.)

Programs were also written to simulate the annually increasing competitional stresses imposed upon each tree by its neighbors as they all grow larger. This was possible for the destructively sampled trees from the three jack pine plots because all of the living and dead trees in each 20 m by 20 m plot had been mapped and described. However, since none of the dead trees had been subjected to TRIM sampling and processing, only surrogate growth could be ascribed to them, based on the average of the growth rates known from the processed trees that had been alive at the time of sampling.

Other programs were written in support of data entry, collation and preliminary examination of monthly summaries of weather and related data from several of the reporting stations closest to the sampling locations. Those summaries usually covered the period from 1951 through 1980. The listings included records for air temperature, total precipitation and rainfall together with estimates of snow melt, potential and actual evapotranspiration and of soil moisture deficits or surpluses. They partially represented the basis for the Climatograms of Ontario, produced by MacIver and Masterston (1984) and the Climatograms by soil type for Ontario, produced by MacIver (1986).

## RESULTS

Before any noteworthy results could be obtained, it was necessary that satisfactory elemental models or equations be found for describing dimensional growth. This exploration began with simple elaborations on the solution of the differential equation called the "law of natural growth or decay". The solution equation is

$$[1] \quad Y_i = Y_0 \times \exp[b \times i]$$

where $\underline{b}$ is a constant rate of growth or decay, depending upon its sign and where $\underline{i}$ ranges from $\underline{1}$ to $\underline{n\text{-}1}$. For $\underline{n}$ years of growth, the first year is labelled year zero; that is, the numbering system goes from $\underline{0}$ to $\underline{n\text{-}1}$ rather than from $\underline{1}$ to $\underline{n}$. This is necessary because every process must begin with a $Y_0$-value greater that 0.0. When [1] is fitted, it integrates the individual increments into a cumulative curve of growth or decay.

[1] has been used to predict the increases in numbers of bacterial cells in certain culture solutions over time and to determine the "half-life" of any radioactive substance. If the b-value is correctly determined, [1] will also give precisely the same results as the equation normally used by banks to calculate the increase to any principal deposited at a fixed rate of interest and compounded annually. For example, the constant b-value for 5% compound interest is the logarithm of 1.05, which is approximately 0.04879016417.

Some of the exploratory fits based on elaborations of [1] produced large R2 values, but the patterns of their residuals indicated a parabolic bias. This could be partially offset by replacing the b-coefficient with a countering function, but the success was only partial.

It soon became apparent that whenever any growth rate is other than constant, as is the case with usually irregular dimensional growth in living organisms, we must move from the realm of differential-based equations to that of difference-based equations. The difference-based equivalent of the integrating term for accumulation in a differential-based equation is a summating term. Dimensional growth in trees usually proceeds at rates that irregularly fluctuate between increases and decreases and that tend to diminish over time. Table 1 illustrates this. It presents 126 values for growth of dbhob in a hard maple tree. It also presents 125 values for annual growth rates and 124 values for differences in rates, of which 69 or about 56% were negative.

To calculate the rate for any year of growth, one simply takes the natural logarithm of the quantity total growth at the end of that year divided by total growth at the end of the previous year. The values in Table 1 were abbreviated to save space, but if the rates are calculated to, say, 12 or more significant digits, it will be seen that

$$[2] \quad Y_t = Y_0 \times \exp\left[\sum_{i=1}^{t}(b_i)\right]$$

is the appropriate equation for accumulating growth at varying rates, with the individual b-values identified by subscripts. [2] will permit exact calculation of each growth value after the known 0th if the calculating precision is adequate. This means that the set of 125 rates plus the 0th growth value contains precisely the same information as does the set of 126 growth values.

Trajectories for either growth or decay associated with constantly increasing or decreasing rates may be created by adding a second rate-coefficient to set the changes in the overall growth rate. The original rate from [1] is renamed $c_1$ and the second coefficient is labelled $c_2$. Select any desired values for $Y_0$ and for $c_1$. Select a value for $c_2$ that is larger than 1.0 for an increasing combined rate or smaller than 1.0 for a decreasing one. Calculate subsequent $Y_i$-values by raising c2 to powers that increase from 0 to i-1 and by multiplying each such result by $c_1$. Then

$$[3] \quad Y_i = Y_{i-1} \times \exp[c_1 \times c_2^{i-1}]$$

can be seen to be the correct equation for individual increments and

$$[4] \quad Y_t = Y_0 \times \exp\left[\sum_{i=1}^{t}(c_1 \times c_2^{i-1})\right]$$

can be seen to be the correct summation equation, when both of the coefficients obtained from [3] and applied to [4] were constant over the fitted period. In such cases, the precise relationship between growth and the combined rate can be calculated exactly. [3] and [4] are the bases for the two kinds of residual tables mentioned earlier.

It is necessary to have an equation on the order of [3] if we want to utilize static equations in projecting growth patterns over time. [2] describes what has gone before but contains nothing to indicate trends or on which to base projections.

It was initially intended that six different dimensions of tree growth would be investigated for each tree, namely: dbhob, basal area derived from that diameter, total height, total area of the bole cambial layer between wood and bark, gross total volume and gross merchantable volume. However, the results of fitting [3] to the first two of those dimensions provided coefficients so closely related that basal area data were not routinely fitted. The $c_1$-values derived from fitting basal areas were almost exactly twice the size of those derived from fitting dbhob's; the two sets of

$c_2$-values were almost identical. The comparisons usually were in agreement to six or seven places, occasionally to five or eight places, regardless of the length or location of the fitted period.

Strictly speaking, whenever fitting of [3] is mentioned, it should be understood that the version of [3] actually fitted was

$$b_i = c_1 \times c_2^{i-1}.$$

Figure 1 illustrates graphs of the $b_i$-values for the six dimensions of a dominant jack pine tree.

Fitting [3] for five-year periods to any of the dimensions of all three species tended to give from good to excellent results except for the first few periods. Applying those fitted coefficients in [4] usually led to larger $R^2$ values and to smaller differences between the terminal growth value and its estimate than were obtained by the fitting. Exceptions to this usually were confined to the first period or two. Almost all of the terminal deviations in [4] were less than 1.0% after the first or second period and most became less than 0.1% soon after that, with many of them much smaller.

Fitting [3] for periods of increasing length tended to give results that were successively slightly poorer. Graphs of fits from years 1 through n-1 can be seen in Figure 2 for a codominant hard maple tree, in Figure 3 for the dominant jack pine tree and in Figure 4 for the codominant black spruce tree. Please note that Figures 2-4 tend to represent the poorest fits that were obtained for those trees. The longest periods tended to have the poorest fits in most dimensions for any tree of all three species and application of the resulting coefficients in [4] often led to such low $R^2$ values that meaningful applications were nonexistent; those values were occasionally less than 0.1 in the worst cases. Such disparities probably result from using uncorrected regression values. However, the older the tree, the better the whole-life fit using [3]. Graphs of such fits in the "300"-year-old hard maple are not presented; putting 300 symbols on a single sheet caused them to overlap to a unprintable extent. However, the fitted curves for every dimension of that tree each fell entirely with the symbols.

An example of the numeric results of long-period fitting of the dbhob growth in the dominant jack pine tree is given in Table 2. That pattern of results is more or less representative for the dimensions of all of the trees studied, although good results were obtained for certain dimensions of some individual trees for 40 or more years. Increased age tended simply to extend the intermediate period between the usually poor start, eg. the first two values in each of the last two columns of Table 2, and the usually very good end, eg. the last three values, as exemplified by the $R^2$ and the percent-deviation values in the last two columns.

In all three species, fits of dbhob or basal area tended to be slightly better than the others; fits of gross merchantable volume tended to be slightly poorer than the others. Comparable results were also obtained when growth values for destructively sampled trees from any plot were aggregated into a single set of values for a "giant" bole and fitted. Such aggregation permits determination of production by surviving trees per unit area over time.

Weighted regression was explored as a possible means of reducing the disparities between the results of using [3] and [4]. For each fit over any period length, progressively increased weights were assigned from the earliest to the last observed value. This produced trivially better fits over the first half of the second type of scanning and trivially poorer fits over the second half. Its use was not pursued further.

Preliminary results incorporating simulated values for competition from each tree's neighbors appeared to be very promising. However, the lack of information about the dead trees suggested that more complete data are needed before further work in this area would be justified.

Preliminary investigation of the weather records suggested that climatic impacts on the dimensional growth of trees generally result from the dynamic interaction of several factors,

although some years with either very good or very poor growth may be directly associated with threshold values.

## DISCUSSION

Regarding neighborly competition, it appears that sequences of three or more positive differences between rates probably indicate partial releases by the dropping out of near neighbors. Examples of such sequences can be seen in Table 1 for the periods 1868 to 1870, 1887 to 1889, 1906 to 1909, 1914 to 1916, 1931 to 1934, and 1978 to 1981 in the dbhob growth of a hard maple tree. The period from 1946 to 1950 may also suggest the same thing even though there is a negative difference in the middle.

Such sequences were rarely found in height data but were commonly found in all the other dimensions, especially in the hard maple. Those trees probably were all from multistoried stands of uneven age containing mixtures of species which may have experienced frequent mortality. The mortality of hard maple in Algonquin Park appears to have been slightly less during the last three decades investigated than that of the other species with which it is associated in hardwood and mixedwood stands.

The association between such sequences late in the life of jack pine trees and the nearness of standing dead neighbors appears to corroborate the relationship of positive sequences with neighborly mortality although it is not known exactly when those trees died. Such sequences were found less often in the jack pine trees from an even-aged fire stand than in the hard maple and were rare in the black spruce tree from a stand of mixed black spruce and tamarack.

That spruce tree exhibits what may be other sorts of improved growth. It can be seen from Figure 4 that its height growth had been steadily better than average from 1950 to 1971. As indicated earlier, the stand is believed to have undergone considerable spruce budworm damage prior to 1950[2] Fayle and MacIver (1986) pointed out that budworm damage is reflected as abnormalities in the crown portion of growth layer profiles. Figure 5 exhibits some of the growth layer profiles for that tree. It can be seen that the upper portions of the profiles for 1944 and for 1946 through 1949 were almost completely flattened. Apparently the period of recovery from defoliation, begun in 1950, may have been complemented and possibly extended by the benefit of drainage, presumed to have become effective after 1961. No distinct sequence of positive differences in growth rates was found in the listing for any dimension during the period from 1950 through 1971. This suggests that, as is the case with fertilization, draining the excess water from a moderately dense stand will not provide striking improvement unless it is combined with thinning.

The use of difference or differential-difference equations and derivations of them in biological modeling is not new. Watt (1966) mentioned them, Mott (1966) applied them to the modeling of forest insect populations, while Paulik and Greenough (1966) not only applied them to modeling the management of salmon (*Oncorhynchus* spp.) resources but also appeared to recommend employment of the computer simulation language Dynamo that supports their use. More recently, Jayaraman and Bailey (1988) and Gale and Grigal (1988) employed difference or difference-based equations for site indices; Sievänen, Burk and Ek (1988) used them in modeling relationships between photosynthate accumulation and tree dimensions. The present paper may represent the first attempt to employ a difference-based equation as a static elemental model for tracking growth in various dimensions of any tree bole.

The suitability of difference-based equations when applied to measurements associated with fluctuating growth rates can be confirmed by examining [2] for any set of real or generated data. [2] should provide an excellent basis for evaluating the impacts of various factors influencing tree growth. The exactness of [2] is limited only by the precision of the calculations and the accuracy of the measurements.

Similar examinations of [3] and [4] will demonstrate equal exactness if the growth rate is undergoing constant change. It might be argued that the use of [3] is less defensible because it indicates an overall trend of diminishing growth rates while Table 1 demonstrates that the differences between rates may be positive, and thus increasing, almost half of the time. However, [3] may be useful for when smoothing is required, because it "tracks" the changing growth trajectories fairly well; also, it can provide a basis for projection. It might provide a useful starting place in the development of a dynamic model and in the generation of improved "proxy" data employed by the people responsible for replacing missing weather records.

The changes of growth trajectories may tend to become slightly less significant as the trees get larger. During the last 30 to 40 years before cutting, dimensional growth in all of the trees tended to follow an increasingly fixed trajectory with small deviations. In the jack pine, for example, the sequences of positive differences in rates associated with partial releases late in their lives through neighbor mortality appear to reflect smaller deviations than similar releases that had occurred earlier. This tendency appears to offer the small consolation that predictions begun when the stands are nearly mature could remain fairly accurate for a few decades.

Fitting [3] and using [4] to test the usefulness of the result should provide a better basis for prediction than can be obtained by fitting existing differential-based equations. Any candidate period of growth for which the $R^2$-value from [4] approaches or exceeds 0.9999 should be satisfactory. However, even if this proves to be the case, we will have solved only a very small part of the overall problem. Clearly, whenever we base an extended prediction on a recent trajectory at any time, we must expect the possibility that future growth may deviate from that trajectory, thus increasing the error in the prediction.

Paulik and Greenough (1966) said of the salmon management problem, "No matter how elaborate and how refined these computerized versions of traditional analytical models become, they still remain static models of dynamic processes and their usefulness is completely dependent upon the manager's ability to forecast the future." They were dealing with a situation where a good dynamic model would solve their problems by supporting almost day to day adjustments to their management procedures. Our prediction problems can be solved with a good dynamic model only if it supports the making of year to year adjustments to our static growth and yield predictions. That is, we need dynamic predictions capable of accepting and responding to the variations in growth rates which may occur any year in apparent reaction of the growth processes to the varying influences of changing combinations of environmental factors. Once we have made a prediction, we appear to have a choice between two alternatives for making such corrections.

The first alternative would not require us to learn anything about the growth processes or to develop a dynamic model. All we would need to do is to establish an appropriate number of permanent sample plots in every stand, on which annual, continuous forest inventory would be practiced until the stands became mature. At any time after the first six years of measurement, either individual or aggregated trees can be fitted with [3] and tested with [4]; if there is little disparity between those results, then the coefficients from fitting can be used to predict future growth. The known growth and damage can be utilized to update the predictions each year. The cost of this alternative would almost certainly be prohibitive unless it were confined to small areas designated as representing "prime" sites, for example. It most likely would not provide a sound basis for evaluating or explaining the degrees of primeness.

The second alternative, initially expensive but almost certainly cheaper in the long run, would be to learn how to identify and to evaluate the effective environmental factors so that, eventually, monitoring of the environment, the mortality and the damage could substantially replace monitoring of the growth of standing trees. Furthermore, this alternative may hold the only hope we have for ever being able to evaluate site quality correctly or to determine the impact of atmospheric pollution on productivity.

With either alternative, the rates of either dbhob or basal-area growth per unit area may prove important in estimating volume increments. This is not equivalent to choosing one or more trees as representing mean basal area and multiplying their mean volume by the frequency.

Decreases in growth rates can have many causes. The most persistent of these would appear to be competition, although it is possible that close association with neighbors during the early years may provide a beneficial or "nursing" effect. Other causes exist in physical damage such as may arise from insect infestations, ice or wind storms and early fall or late spring frosts. Such frosts appear to be associated with what MacIver (1988) calls "thermal stress" periods, although the periods appear to be those during which annual temperatures fluctuate about the trends more markedly than usual; he selected species-spacing, soils and thermal stress as the critical factors in growth and development processes. Still other causes may lie in physical or chemical degradation of site quality; for example, through the development of "pans" in the soil, flooding of the site, exhaustion of nutrients or deposition of airborne contaminants. Frequently occurring small diminutions appear most likely to be caused by poorer-than-average climatic years for growth.

Increases in growth rates appear to have fewer causes. Good climatic years for growth may be the most common of these, although simple recovery from poor years or from previous damage may not be uncommon. Site improvements such as fertilization or drainage of flooded areas appear likely to bring about at least temporary increases. The greatest and most persistent increases may be those that develop when any tree is partially released as the result of either artificial or natural thinning. These appear to be marked by sequences of positive differences in growth rates. Any sequence of three or more positive differences appears to indicate such events; shorter sequences might be found to be associated with losses of neighbors that were either smaller or farther away. If the second alternative is to be pursued, then it is strongly recommended that we begin by improving the data collection and compilation procedures in support of future analyses. The within-tree sampling intensities indicated by MacIver, Raymond and Masarovich (1988) should be understood to represent compromises between analytical needs and operational acceptance.

Every tree within each 10 m by 10 m area designated for destructive sampling must be sectioned from top to ground level. This means every tree whether living or dead, not just the living trees of those species of commercial interest. We need the growth information from the dead trees for evaluating competitional stresses.

A new algorithm for height interpolations is necessary. Whenever the bolt between any two disks contains more than two annual heights, the present procedure assumes equal spacings between them. In order to improve upon the accuracy and precision of interpolating annual heights, disks should be taken at shorter intervals. A special study should be undertaken in which both exterior and interior determinations of those heights are made. Intervals between disks should be allowed to vary so that, if possible, they are not cut at nodal points.

Also, the adequacy of measuring ring widths from only two radii per disk, starting from north and south points, should be re-examined. Initially, at least four and perhaps eight radii should be measured in each disk for analytical purposes. If the average of only two radii is to be utilized for operational purposes, then the possibility that the most appropriate pair might consist of the longest and the shortest radius in each disk may be worthy of consideration. What we wish to obtain is the set of averaged ring widths that most accurately represent the area encompassed by each successive ring.

It is hoped that the foregoing intensifications of within-tree sampling done in support of further analyses will prove to provide more disks and more measured radii than are needed for that purpose. If, and only if, that happens will we have a sound basis for determining both the cost savings and the error increases associated with each step in relaxing the requirements toward operational acceptability.

For analytical purposes, at least, the compilation software should be revised so that all computations are done and the results stored using double-precision arithmetic.

When all of the foregoing recommendations have been implemented in support of sampling from a few thoughtfully and carefully selected stands, then we will be able to begin meaningful analyses for the purpose of identifying and evaluating those environmental factors influencing growth. The analyses should consider those factors to be acting and interacting jointly.

A comment on regression. Usually when regression is applied, the fitting accounts for a certain proportion of the total variation in the values of a dependent variable regressed on a set of "independent" variables. That accounting is commonly said to "explain" the determined proportion of variation. In most biological cases it explains nothing more than the usually "confounded" numeric correlations that were present. The only way in which we can ever hope to truly explain or understand dynamic growth processes is through the development and application of dynamic growth models; then we can begin to make increasingly sound dynamic predictions of growth and yield. It is hoped that the present paper may help to encourage studies in those domains.

## PROGRAM AVAILABILITY

An IBM-compatible version of the Fortran source code for the Marquardt-Algorithm regression package developed for equations [3] and [4] can be obtained by sending an IBM-compatible diskette to:

Frank Raymond Consulting
24 Royal Road
Aurora, Ontario L4G 1A8
Phone: (416) 727-5715.

## ACKNOWLEDGMENTS

The author would like thank D. L. McLaughlin, Ontario Ministry of Environment, for the hard maple data; D. C. MacIver, Atmospheric Environment Service, for the weather records; S. J. Masarovich, Ontario Ministry of Natural Resources, and D. C. MacIver for the conifer data; F. O. J. Matejka, D. N. Bates and D. L. Gillmeister, Ontario Tree Improvement and Forest Biomass Institute, for providing advice and the use of their computer facilities in the preparation of this manuscript.

## REFERENCES

BELSLEY, D. A., E. KUH and R. E. WELSCH. 1980. Regression diagnostics: identifying influential data and sources of collinearity: J. Wiley and Sons, Inc., N.Y. 292 pp.

CUNIA, T. 1964. Weighted least squares method and construction of volume tables. For. Sci. 10: 180-191.

DRAPER, N. R. and H. SMITH. 1966. Applied regression analysis. John Wiley and Sons, Inc., New York. 407 pp.

DEMAERSCHALK, J. P. and A. KOZAK. 1977. The whole-bole system: a conditioned dual-equation system for precise prediction of tree profiles. Can. J. For. Res. 7: 488-497.

FAYLE, D. C. F. and D. C. MACIVER. 1986. Growth layer analysis as a method of examining tree growth and development responses. *In* Environmental influences on tree and stand increment.

Edited by D. S. Solomon and T. B. Brann. IUFRO, 1985, Maine Agric. Expt. Sta., Univ. Maine Misc. Publ. 691, pp. 40-48.

GALE, M. R. and D. F. GRIGAL. 1988. Performance of a soil productivity index model used to predict site quality and stand production. *In* Forest growth modelling and prediction. Edited by A. R. Ek, S. R. Shifley and T. E. Burk. IUFRO, 1987, N. Cen. For. Expt. Sta., Saint Paul, Minn. Gen. Tech. Rpt. NC-120, 1: 403-410.

GROSENBAUGH, L. R. 1967. REX--Fortran-4 system for combinatorial screening or conventional analysis of multivariate regressions. USDA For. Serv. Res. Pap. PSW-44. 47 pp.

JAYARAMAN, K. and R. L. BAILEY. 1988. Modelling mixed species stands with Scheffe's canonical polynomials. *In* Forest growth modelling and prediction. Edited by A. R. Ek, S. R. Shifley and T. E. Burk. IUFRO, 1987, N. Cen. For. Expt. Sta., Saint Paul, Minn. Gen. Tech. Rpt. NC-120, 1: 201-208.

MACIVER, D. C. 1986. Climatograms/soil type for Ontario. For. Res. Group, Min. Nat. Res., Mimeographed., 7 pp.

------- 1989. Thermal stress in the new forest. *In* Proceedings of climate applications in forest renewal and forest production. 1986. Edited by D. C. MacIver, R. B. Street and A. N. Auclair. Orillia, Ontario. pp. xxx-xxx.

------- and J. MASTERSTON. 1984. Climatograms of Ontario. *In* Proceedings of the Workshop on evapotranspiration, irrigation and plant moisture stress. 1984. Edited by D. C. MacIver and R. B. Street. Toronto, Ontario. pp. 49-55.

-------, F. L. RAYMOND and S. J. MASAROVICH. 1989. Physio-climate detection and modelling with the computerized tree ring increment measuring (TRIM) system. *In* Proceedings of climate applications in forest renewal and forest production. 1986. Edited by D. C. Maciver, R. B. Street and A. N. Auclair. Orillia, Ontario. pp. xxx-xxx.

MOTT, D. G. 1966. The analysis of determination in population systems. *In* Systems analysis in ecology. Edited by K. E. F. Watt. Academic Press, New York and London. pp. 179-194.

PAULIK, G. J. and J. W. GREENOUGH, JR. 1966. Management analysis for a salmon resource system. *In* Systems analysis in ecology. Edited by K. E. F. Watt. Academic Press, New York and London. pp. 215-252.

RAYMOND, F. L. 1980. INREG--an interactive computer program for multiple linear regression by least squares. Ont. Min. Nat. Res., For. Res. Rpt. No 106, 21 pp.

SIEVANEN, R., T. E. BURK and A. R. EK. 1988. Parameter estimation in a photosynthesis-based growth model. *In* Forest growth modelling and prediction. Edited by A. R. Ek, S. R. Shifley and T. E. Burk. IUFRO, 1987, N. Cen. For. Expt. Sta., Saint Paul, Minn. Gen.Tech. Rpt. NC-120, 1: 345-352.

WATT, K. E. F. 1966. The nature of systems analysis. *In* Systems analysis in ecology. Edited by K. E. F. Watt. Academic Press, New York and London. pp. 1-14.

TABLE 1. Growth values, rates and differences for diameters outside bark at breast height, for the hard maple tree.

| i | Dbhob$_i$[a] | $b_i$ | Diff. | Year | i | Dbhob$_i$[a] | $b_i$ | Diff. | Year |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.32424 | | | 1860 | 54 | 9.94262 | 0.01655 | 0.00320 | 1914 |
| 1 | 0.51460 | 0.46191 | | 1861 | 55 | 10.12461 | 0.01814 | 0.00159 | 1915 |
| 2 | 0.64429 | 0.22477 | -0.23714 | 1862 | 56 | 10.34007 | 0.02106 | 0.00292 | 1916 |
| 3 | 0.79072 | 0.20479 | -0.01997 | 1863 | 57 | 10.55344 | 0.02043 | -0.00063 | 1917 |
| 4 | 0.90368 | 0.13353 | -0.07126 | 1864 | 58 | 10.74589 | 0.01807 | -0.00235 | 1918 |
| 5 | 1.03547 | 0.13613 | 0.00260 | 1865 | 59 | 10.92788 | 0.01679 | -0.00128 | 1919 |
| 6 | 1.15680 | 0.11080 | -0.02533 | 1866 | 60 | 11.09523 | 0.01520 | -0.00160 | 1920 |
| 7 | 1.27185 | 0.09482 | -0.01598 | 1867 | 61 | 11.37763 | 0.02513 | 0.00994 | 1921 |
| 8 | 1.40782 | 0.10157 | 0.00675 | 1868 | 62 | 11.68932 | 0.02703 | 0.00189 | 1922 |
| 9 | 1.61283 | 0.13594 | 0.03437 | 1869 | 63 | 11.94453 | 0.02160 | -0.00543 | 1923 |
| 10 | 1.85966 | 0.14241 | 0.00647 | 1870 | 64 | 12.16208 | 0.01805 | -0.00355 | 1924 |
| 11 | 2.00191 | 0.07371 | -0.06870 | 1871 | 65 | 12.38382 | 0.01807 | 0.00002 | 1925 |
| 12 | 2.18181 | 0.08605 | 0.01235 | 1872 | 66 | 12.59091 | 0.01658 | -0.00148 | 1926 |
| 13 | 2.36799 | 0.08188 | -0.00417 | 1873 | 67 | 12.81474 | 0.01762 | 0.00104 | 1927 |
| 14 | 2.58136 | 0.08628 | 0.00439 | 1874 | 68 | 13.02602 | 0.01635 | -0.00127 | 1928 |
| 15 | 2.79473 | 0.07942 | -0.00686 | 1875 | 69 | 13.24776 | 0.01688 | 0.00053 | 1929 |
| 16 | 3.01019 | 0.07427 | -0.00515 | 1876 | 70 | 13.43393 | 0.01396 | -0.00292 | 1930 |
| 17 | 3.21310 | 0.06523 | -0.00903 | 1877 | 71 | 13.63684 | 0.01499 | 0.00104 | 1931 |
| 18 | 3.40973 | 0.05940 | -0.00583 | 1878 | 72 | 13.96108 | 0.02350 | 0.00851 | 1932 |
| 19 | 3.60637 | 0.05607 | -0.00333 | 1879 | 73 | 14.36481 | 0.02851 | 0.00501 | 1933 |
| 20 | 3.82811 | 0.05967 | 0.00360 | 1880 | 74 | 14.90660 | 0.03702 | 0.00851 | 1934 |
| 21 | 4.08122 | 0.06403 | 0.00436 | 1881 | 75 | 15.44630 | 0.03557 | -0.00146 | 1935 |
| 22 | 4.31760 | 0.05630 | -0.00772 | 1882 | 76 | 15.94417 | 0.03172 | -0.00384 | 1936 |
| 23 | 4.47449 | 0.03569 | -0.02061 | 1883 | 77 | 16.31861 | 0.02321 | -0.00851 | 1937 |
| 24 | 4.62720 | 0.03356 | -0.00213 | 1884 | 78 | 16.76836 | 0.02719 | 0.00397 | 1938 |
| 25 | 4.76526 | 0.02940 | -0.00416 | 1885 | 79 | 17.26204 | 0.02902 | 0.00183 | 1939 |
| 26 | 4.89077 | 0.02600 | -0.00340 | 1886 | 80 | 17.64485 | 0.02193 | -0.00708 | 1940 |
| 27 | 5.02047 | 0.02617 | 0.00017 | 1887 | 81 | 17.87496 | 0.01296 | -0.00898 | 1941 |
| 28 | 5.16690 | 0.02875 | 0.00258 | 1888 | 82 | 18.22221 | 0.01924 | 0.00628 | 1942 |
| 29 | 5.36772 | 0.03813 | 0.00938 | 1889 | 83 | 18.46486 | 0.01323 | -0.00601 | 1943 |
| 30 | 5.53716 | 0.03108 | -0.00705 | 1890 | 84 | 18.57364 | 0.00587 | -0.00735 | 1944 |
| 31 | 5.68986 | 0.02721 | -0.00387 | 1891 | 85 | 18.65941 | 0.00461 | -0.00127 | 1945 |
| 32 | 5.85721 | 0.02899 | 0.00178 | 1892 | 86 | 18.81002 | 0.00804 | 0.00343 | 1946 |
| 33 | 6.04548 | 0.03164 | 0.00265 | 1893 | 87 | 19.03176 | 0.01172 | 0.00368 | 1947 |
| 34 | 6.20864 | 0.02663 | -0.00501 | 1894 | 88 | 19.20747 | 0.00919 | -0.00253 | 1948 |
| 35 | 6.35508 | 0.02331 | -0.00332 | 1895 | 89 | 19.46895 | 0.01352 | 0.00433 | 1949 |
| 36 | 6.51824 | 0.02535 | 0.00204 | 1896 | 90 | 19.77437 | 0.01557 | 0.00204 | 1950 |
| 37 | 6.66885 | 0.02284 | -0.00251 | 1897 | 91 | 20.01702 | 0.01220 | -0.00337 | 1951 |
| 38 | 6.85085 | 0.02692 | 0.00408 | 1898 | 92 | 20.25340 | 0.01174 | -0.00046 | 1952 |
| 39 | 7.03075 | 0.02592 | -0.00100 | 1899 | 93 | 20.57764 | 0.01588 | 0.00414 | 1953 |
| 40 | 7.18973 | 0.02236 | -0.00356 | 1900 | 94 | 20.83285 | 0.01233 | -0.00356 | 1954 |
| 41 | 7.34662 | 0.02159 | -0.00077 | 1901 | 95 | 21.02739 | 0.00929 | -0.00303 | 1955 |
| 42 | 7.54744 | 0.02697 | 0.00538 | 1902 | 96 | 21.25541 | 0.01079 | 0.00149 | 1956 |
| 43 | 7.72524 | 0.02329 | -0.00368 | 1903 | 97 | 21.45204 | 0.00921 | -0.00158 | 1957 |
| 44 | 7.96790 | 0.03093 | 0.00764 | 1904 | 98 | 21.62776 | 0.00816 | -0.00105 | 1958 |
| 45 | 8.15617 | 0.02335 | -0.00757 | 1905 | 99 | 21.90179 | 0.01259 | 0.00443 | 1959 |
| 46 | 8.35489 | 0.02407 | 0.00072 | 1906 | 100 | 22.11934 | 0.00988 | -0.00271 | 1960 |
| 47 | 8.56199 | 0.02448 | 0.00041 | 1907 | 101 | 22.35363 | 0.01054 | 0.00065 | 1961 |
| 48 | 8.78163 | 0.02533 | 0.00085 | 1908 | 102 | 22.63185 | 0.01237 | 0.00183 | 1962 |
| 49 | 9.04312 | 0.02934 | 0.00401 | 1909 | 103 | 22.79711 | 0.00728 | -0.00509 | 1963 |
| 50 | 9.27322 | 0.02513 | -0.00421 | 1910 | 104 | 22.97073 | 0.00759 | 0.00031 | 1964 |
| 51 | 9.47195 | 0.02120 | -0.00392 | 1911 | 105 | 23.14227 | 0.00744 | -0.00015 | 1965 |
| 52 | 9.64976 | 0.01860 | -0.00261 | 1912 | 106 | 23.30961 | 0.00721 | -0.00023 | 1966 |
| 53 | 9.77945 | 0.01335 | -0.00525 | 1913 | 107 | 23.48115 | 0.00733 | 0.00013 | 1967 |

| i | Dbhob$_i$[a] | $b_i$ | Diff. | Year |
|---|---|---|---|---|
| 108 | 23.64850 | 0.00710 | -0.00023 | 1968 |
| 109 | 23.87651 | 0.00960 | 0.00249 | 1969 |
| 110 | 24.05013 | 0.00725 | -0.00235 | 1970 |
| 111 | 24.16519 | 0.00477 | -0.00247 | 1971 |
| 112 | 24.37856 | 0.00879 | 0.00402 | 1972 |
| 113 | 24.61703 | 0.00973 | 0.00094 | 1973 |
| 114 | 24.83458 | 0.00880 | -0.00094 | 1974 |
| 115 | 25.01030 | 0.00705 | -0.00175 | 1975 |
| 116 | 25.08142 | 0.00284 | -0.00421 | 1976 |
| 117 | 25.13162 | 0.00200 | -0.00084 | 1977 |
| 118 | 25.21948 | 0.00349 | 0.00149 | 1978 |
| 119 | 25.39311 | 0.00686 | 0.00337 | 1979 |
| 120 | 25.60439 | 0.00829 | 0.00142 | 1980 |
| 121 | 25.82194 | 0.00846 | 0.00017 | 1981 |
| 122 | 26.01439 | 0.00743 | -0.00104 | 1982 |
| 123 | 26.19220 | 0.00681 | -0.00061 | 1983 |
| 124 | 26.38883 | 0.00748 | 0.00067 | 1984 |
| 125 | 26.55200 | 0.00616 | -0.00132 | 1985 |

[a]Abbreviated values in cm.

TABLE 2. Results of long-period fittings of dbhob growth in the jack pine tree.

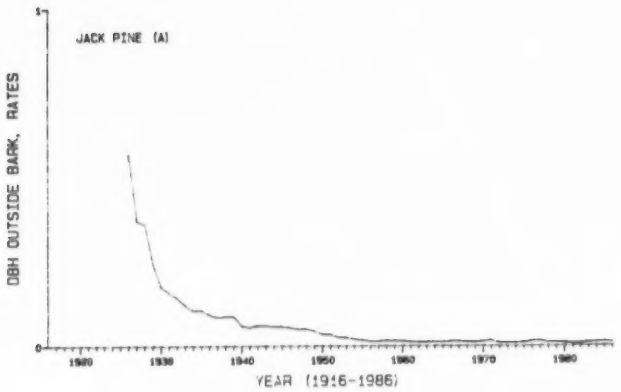
| Years | $c_1$[a] | $c_2$[a] | $Rsq_3$[b] | $Pct_3$[c] | $Rsq_4$[b] | $Pct_4$[c] |
|---|---|---|---|---|---|---|
| 1-61 | 0.4852 | 0.8406 | 0.99973 | 1.015 | 0.81622 | 215.243 |
| 2-61 | 0.3365 | 0.8783 | 0.99984 | 1.000 | 0.92855 | 161.255 |
| 12-61 | 0.0875 | 0.9351 | 0.99997 | 0.686 | 0.99908 | 4.899 |
| 22-61 | 0.0416 | 0.9414 | 0.99997 | 0.618 | 0.99919 | 3.551 |
| 32-61 | 0.0142 | 0.9886 | 0.99999 | 0.005 | 0.99999 | 0.001 |
| 42-61 | 0.0132 | 0.9854 | 0.99999 | 0.013 | 0.99999 | 0.001 |
| 52-61 | 0.0125 | 0.9684 | 0.99999 | 0.075 | 0.99999 | 0.007 |

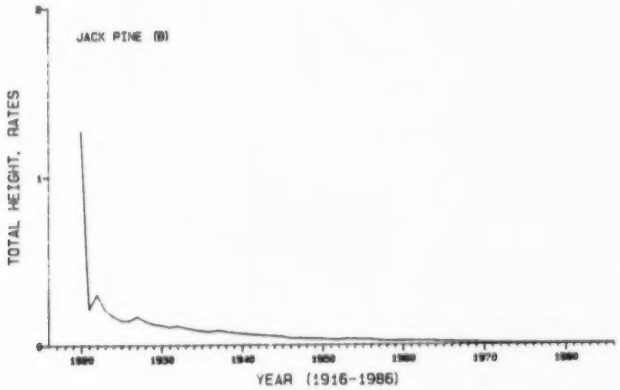
[a] Abbreviated values of the c-coefficients obtained by fitting equation [3].

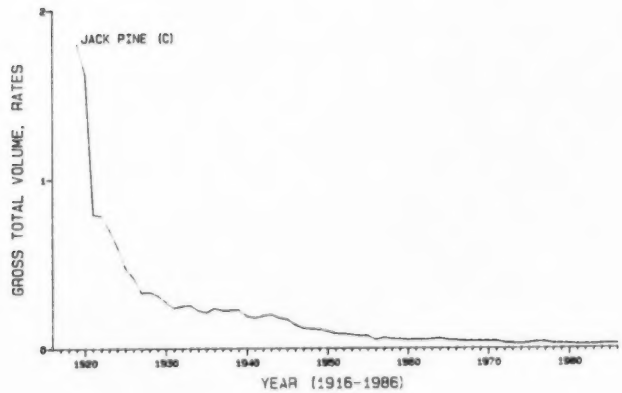
[b] $R^2$ values from equations [3] and [4], respectively.

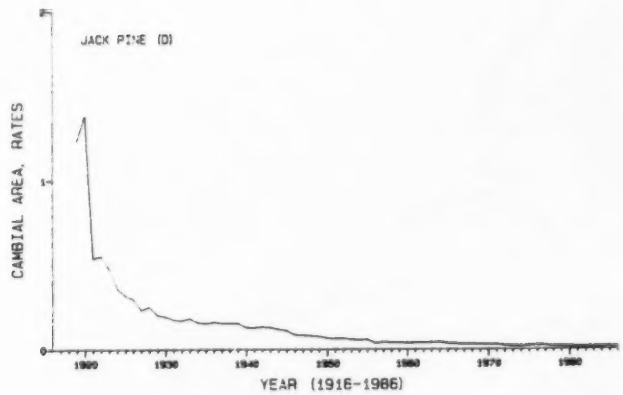
[c] Percentage deviation of the terminal estimate from the terminal observation for [3] and [4], respectively.

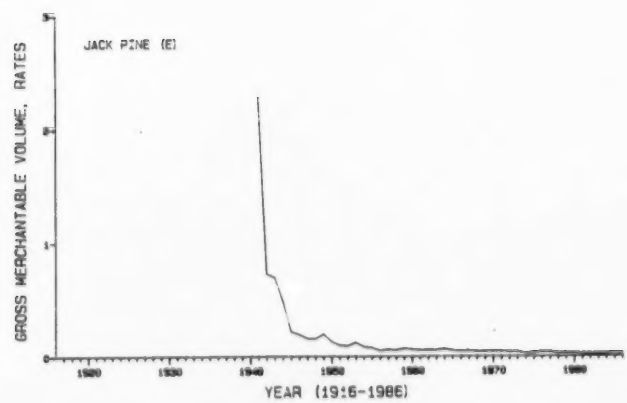
FIG. 1. Plotted b-values for the jack pine tree, as in [2]. (A) is dbhob, (B) is total tree height, (C) is gross total volume, (D) is bole cambial area, (E) is gross merchantable volume and (F) is basal area.



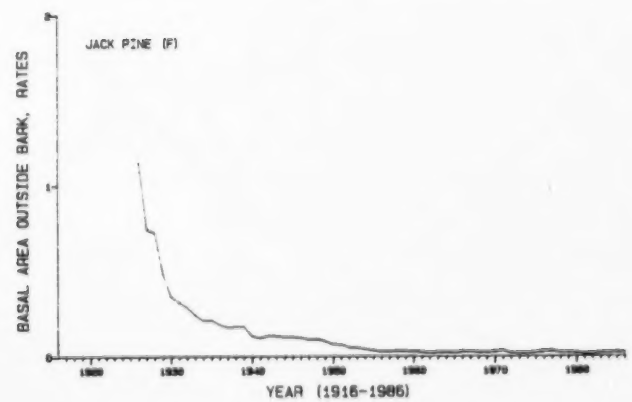
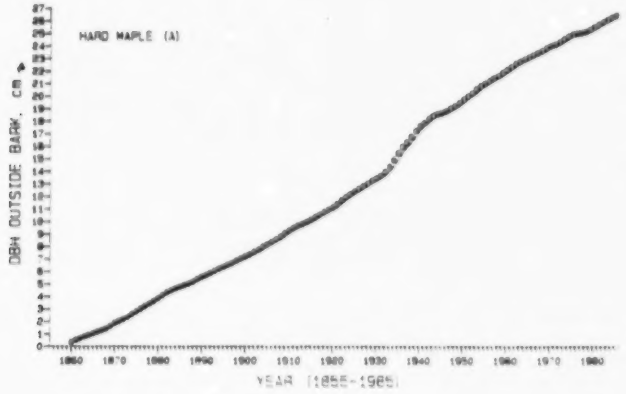
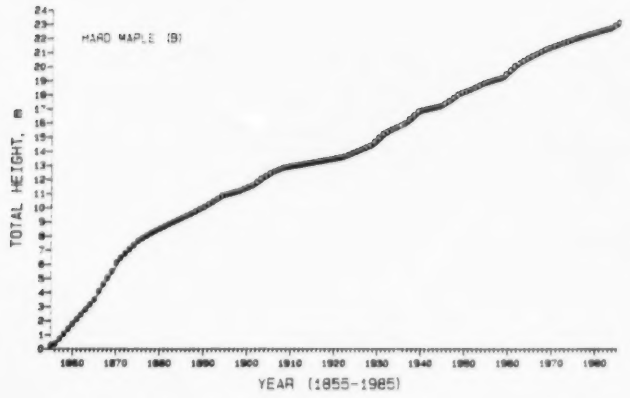
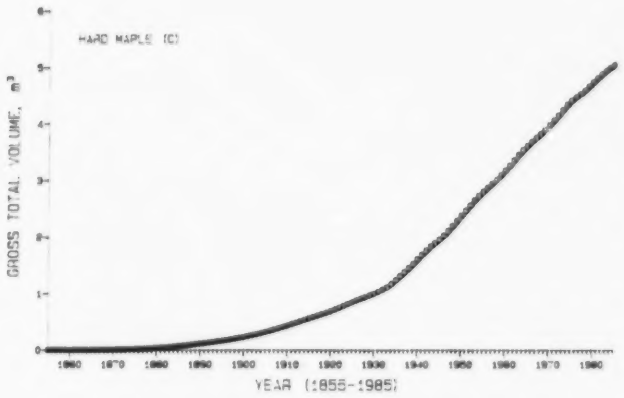
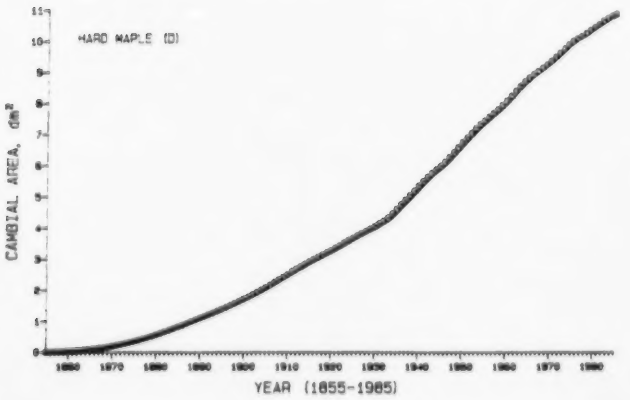



FIG. 2. Growth curves for a 131-year-old codominant hard maple tree, consisting of observed values, o, and fitted curves, --, from [3]. (A) is dbhob, (B) is total tree height, (C) is gross total volume, (D) is bole cambial area and (E) is gross merchantable volume.



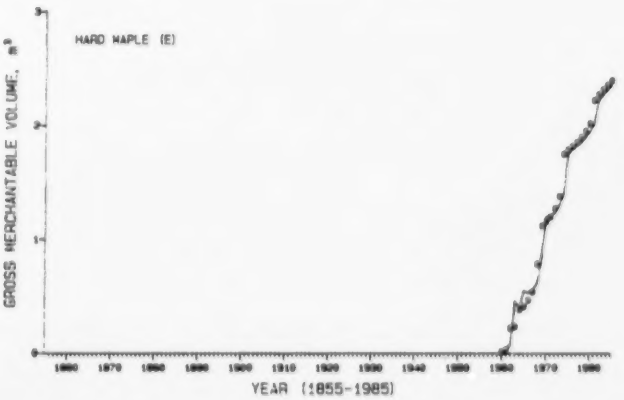
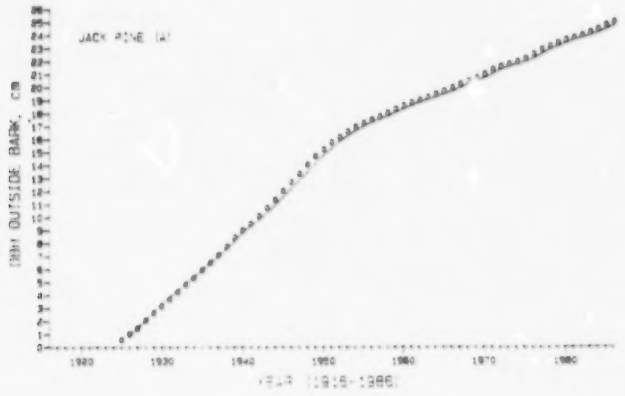
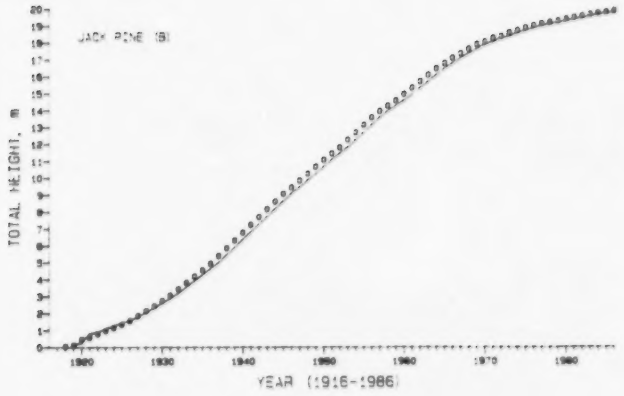
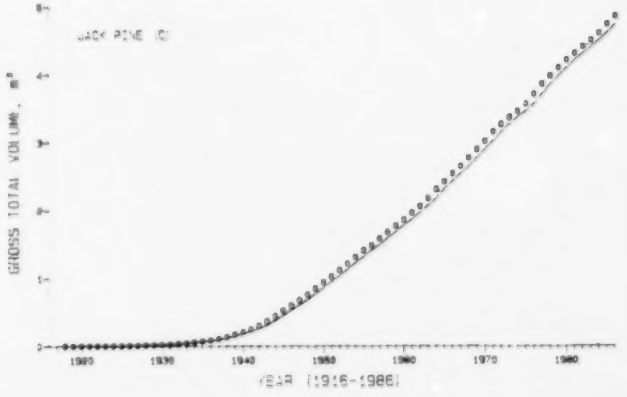
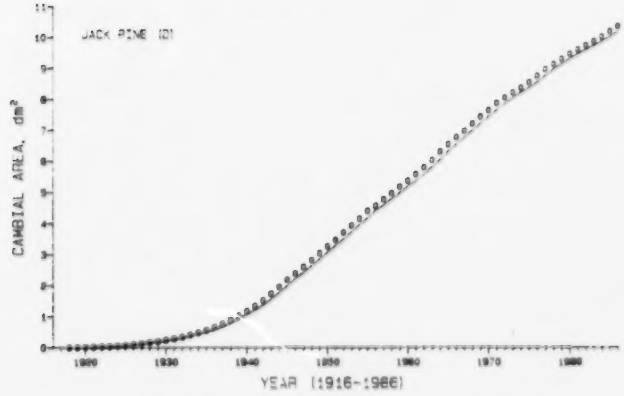



FIG. 3. Growth curves for a 69-year-old dominant jack pine tree, consisting of observed values, o, and fitted curves, --, from [3]. (A) is dbhob, (B) is total tree height, (C) is gross total volume, (D) is bole cambial area and (E) is gross merchantable volume.



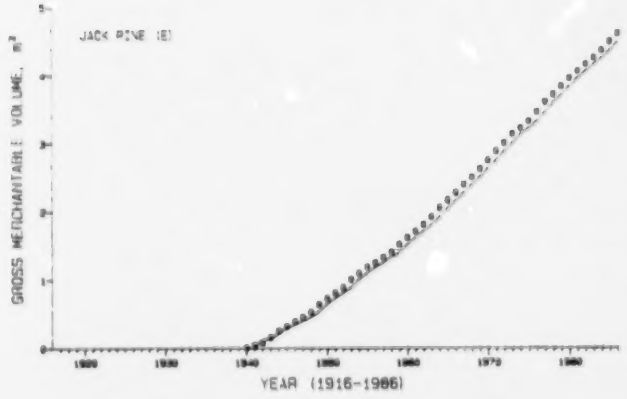
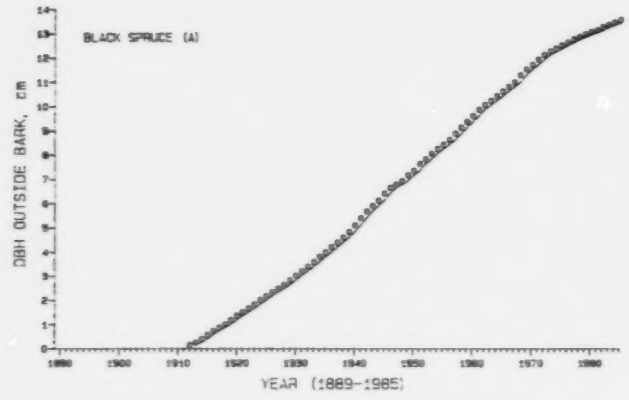
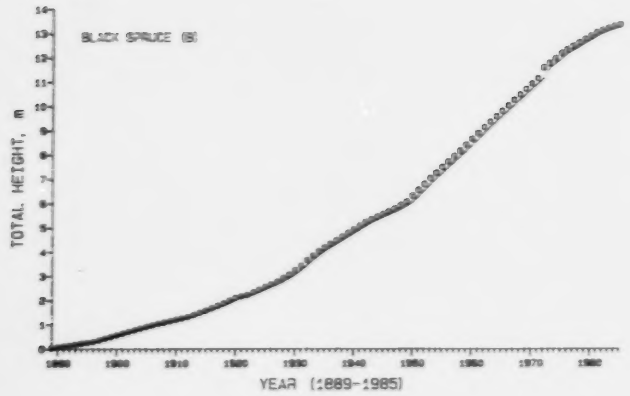
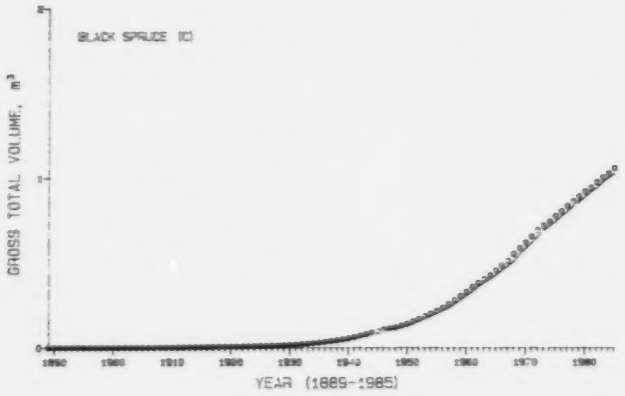
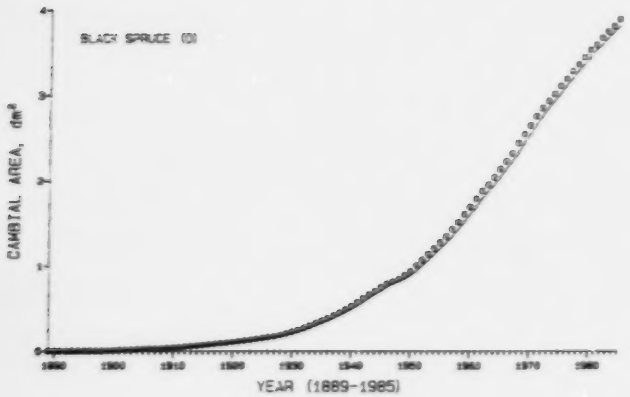



FIG. 4. Growth curves for a 97-year-old codominant black spruce tree, consisting of observed values, o, and fitted curves, --, from [3]. (A) is dbhob, (B) is total tree height, (C) is gross total volume, (D) is bole cambial area and (E) is gross merchantable volume.



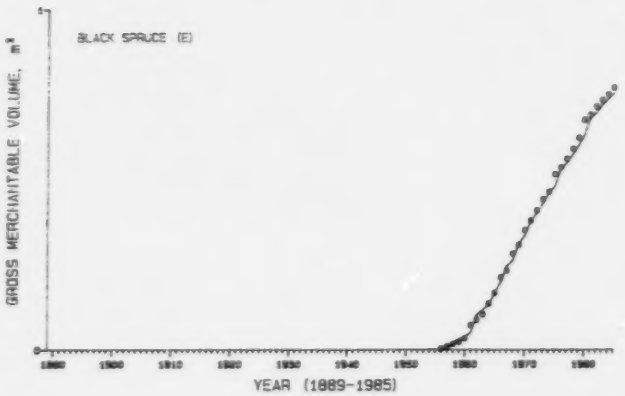



FIG. 5. Growth layer thickness profiles for the last 44 years of growth in the black spruce tree. (A) is for the years 1964 through 1985 and (B) is for the years 1942 through 1963. The short vertical line attached to the top of each curve is a grid marker rather than a part of the curve.